AMC 10 Student Practice Questions

You will find these and additional problems for the AMC 10 and AMC 12 on AMC's web site: amc.maa.org, available from the current and previous AMC 10/12 Teacher Manuals, amc.maa.org/amc/e-exams/e6-amc12/archive12.shtml) or from our Problems page archives (amc.maa.org/amc/a-activities/a7-problems/problem81012archive.shtml).

A ferry boat shuttles tourists to an island every hour starting at 9 AM until its last trip starting at 3 PM. One day the boat captain notes that on the 9 AM trip there were 100 tourists on the ferry boat, and that on each successive trip, the number of tourists was 1 fewer than on the previous trip. How many tourists did the ferry take to the island that day?

(A) 585 **(B)** 594 **(C)** 672 **(D)** 679 **(E)** 694

2010 AMC 10 A, Problem #3—
2010 AMC 12 A, Problem #2—
"The ferry boat makes 7 trips to the island everyday."

Solution

Answer (D): The ferry boat makes 7 trips to the island. The number of tourists shuttled was

$$\begin{aligned} 100 + (100-1) + (100-2) + (100-3) + (100-4) + (100-5) + (100-6) \\ &= 7 \cdot 100 - (1+2+3+4+5+6) \\ &= 700 - 21 \\ &= 679. \end{aligned}$$

Difficulty: Easy

NCTM Standard: Number and Operations Standard for Grades 9–12: compute fluently and make reasonable estimates.

Point $A = (4,3)$ is rotated counterclockwise $90°$ around the origin to point A'. The point A' is reflected across the x-axis to point A''. What is the distance between A and A''?

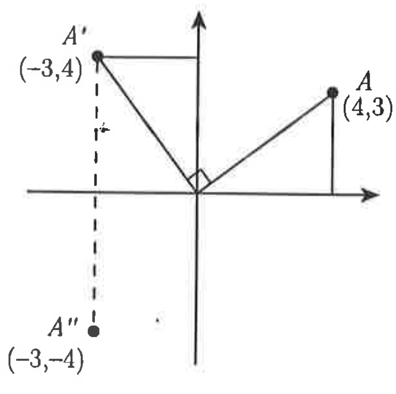
(A) $\sqrt{2}$ **(B)** 8 **(C)** $7\sqrt{2}$ **(D)** 10 **(E)** 14

2010 AMC 10 A, Problem #6— "Visualize the transformations by drawing."

Solution

Answer (C): The figure shows that $A' = (-3,4)$ and $A'' = (-3,-4)$. The distance between A and A'' is

$$\sqrt{(4-(-3))^2 + (3-(-4))^2} = \sqrt{7^2+7^2} = 7\sqrt{2}.$$



Difficulty: Medium-easy

NCTM Standard: Geometry Standard for Grades 9–12: apply transformations and use symmetry to analyze mathematical situations.

In a certain town bicycle license plates are to have 3 different letters, each letter chosen from A through F, followed by 3 different digits, each digit chosen from 1 through 6. The letters will appear in alphabetical order and the digits in numerical order. For example, $ACE236$ is allowed and $ADB124$ is not allowed. How many such license plates are possible?

(A) 36 **(B)** 72 **(C)** 120 **(D)** 240 **(E)** 400

2010 AMC 10 A, Problem #13—

"There are $\binom{6}{3} = 20$ letter combinations to choose from."

Solution

Answer (E): There are

$$\binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$$

letter combinations to choose from, and each particular choice results in exactly one possible configuration for the letters in a license plate. Similarly, there are $\binom{6}{3} = 20$ possible configurations for the digits. Hence there are $20 \cdot 20 = 400$ possible license plates.

Difficulty: Medium

NCTM Standard: Algebra Standard for Grades 9–12: understand relations and functions and select, convert flexibly among, and use various representations for them.

Three children have a foot race in which the leader after 1 minute is declared the winner. Each child runs at a constant speed. At the end of the minute, Alicia has run 720 feet. Ben, who started 40 feet in front of Alicia, has run 600 feet from his starting point. Cheryl, who started 40 feet in front of Ben, has run 400 feet from her starting point. For how many seconds during the race was Ben in the lead?

(A) 8 **(B)** 10 **(C)** 12 **(D)** 15 **(E)** 20

2010 AMC 10 A, Problem #16—
2010 AMC 12 A, Problem #9—
"Use inequalities to represent the relationships."

Solution

Answer (A): The distance of Alicia, Ben, and Cheryl from Alicia's starting point after t minutes are, respectively, $720t$, $40 + 600t$, and $80 + 400t$ feet. For Ben to be in the lead we must have

$$40 + 600t > 720t \quad \text{and} \quad 40 + 600t > 80 + 400t.$$

These inequalities are equivalent to

$$40 > 120t \quad \text{and} \quad 200t > 40$$

or

$$\frac{1}{3} > t \quad \text{and} \quad t > \frac{1}{5}.$$

Therefore Ben is in the lead when $\frac{1}{3} > t > \frac{1}{5}$, a time interval of $\frac{1}{3} - \frac{1}{5} = \frac{2}{15}$ minutes, or $\frac{2}{15} \cdot 60 = 8$ seconds.

Difficulty: Medium-hard

NCTM Standard: Algebra Standard for Grades 9–12: write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency-mentally or with paper and pencil in simple cases and using technology in all cases.

Suppose that $ABCD$ is a square, M is the midpoint of $\overline{AB}$, and E is the intersection of $\overline{CM}$ and $\overline{BD}$. What is $\frac{EB}{EM}$?

(A) 1 **(B)** $\frac{5}{4}$ **(C)** $\frac{2\sqrt{10}}{5}$ **(D)** $\frac{\sqrt{15}}{3}$ **(E)** $\frac{4}{3}$

2010 AMC 10 A, Problem #18—
"Let F be the foot of the altitude to $\overline{AB}$ from E."

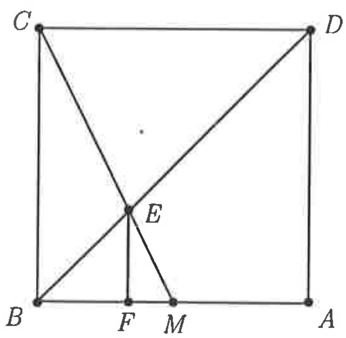
Solution

Answer (C): Let F be the foot of the altitude to $\overline{AB}$ from E and let $EF = x$. Then $EB = \sqrt{2}x$ because $\triangle BEF$ is a $45-45-90°$ triangle. Triangle EMF is similar to triangle CMB, so $FM = \frac{x}{2}$. By the Pythagorean Theorem applied to $\triangle EMF$,

$$EM = \sqrt{x^2 + \left(\frac{x}{2}\right)^2} = \frac{\sqrt{5}}{2}x.$$

Thus

$$\frac{EB}{EM} = \frac{\sqrt{2}x}{\frac{\sqrt{5}}{2}x} = \frac{2\sqrt{2}}{\sqrt{5}} = \frac{2\sqrt{10}}{5}.$$



Difficulty: Hard

NCTM Standard: Geometry Standard for Grades 9–12: analyze properties and determine attributes of two- and three-dimensional objects.

AMC 12 Student Practice Questions

How many integers in the set $\{1, 2, 3, \ldots, 15\}$ cannot be the degree measure of the exterior angle of a regular polygon?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

2010 AMC 12 B, Problem #7—
"The sum of the degree measure of the exterior angles of a convex polygon is 360."

Solution

Answer (A): The sum of the degree measure of the exterior angles of a convex polygon is 360, so the degree measure of each exterior angle of a regular n-gon is $\frac{360}{n}$ for $n \geq 3$. This is a positive integer k precisely when k is a factor of 360 and $k \leq 120$. For $k \leq 15$, all values are factors of 360 except 7, 11, 13, and 14.

Difficulty: Medium-easy

NCTM Standard: Geometry Standard for Grades 9–12: analyze properties and determine attributes of two- and three-dimensional objects.

Let f be a function such that $f(x-y)=f(x)+y$ for all real numbers x and y and $f(40)=20$. What is $f(30)$?

(A) 10 **(B)** 20 **(C)** 30 **(D)** 40 **(E)** 50

2010 AMC 12 B, Problem #11—

"$f(30)=f(40-10)$."

Solution

Answer (C): Note that

$$f(30)=f(40-10)=f(40)+10=20+10=30.$$

The function $f(z)=-z+60$ satisfies the given conditions.

Difficulty: Hard

NCTM Standard: Algebra Standard for Grades 9–12: understand relations and functions and select, convert flexibly among, and use various representations for them.

Let A and B be two independent events such that the probability that A occurs but B does not is $\frac{1}{4}$, and the probability that B occurs but A does not is $\frac{1}{10}$. There are two possible values, p and q, for the probability that A and B both occur. What is $p+q$?

(A) $\frac{7}{20}$ **(B)** $\frac{9}{20}$ **(C)** $\frac{11}{20}$ **(D)** $\frac{13}{20}$ **(E)** $\frac{3}{4}$

2010 AMC 12 B, Problem #17— "Let c be the probability that A and B both occur."

Solution

Answer (D): Let c be the probability that A and B both occur. Then the probability of A is $c+\frac{1}{4}$, and the probability of B is $c+\frac{1}{10}$. Because A and B are independent, $c=(c+\frac{1}{4})(c+\frac{1}{10})$, so $c^2-\frac{13}{20}c+\frac{1}{40}=0$. The two roots of this quadratic polynomial sum to the negative of the coefficient of c, which is $\frac{13}{20}$.

Note: The two roots are $c=\frac{13\pm\sqrt{129}}{40}$; they are both positive and less than $\frac{3}{4}$.

Difficulty: Hard

NCTM Standard: Data Analysis and Probability Standard for Grades 9–12: understand and apply basic concepts of probability.

In a right triangle, one altitude has length 60, and the sine of one angle is 0.6. What is the largest possible area of this triangle?

(A) 2250 **(B)** 2400 **(C)** 3000 **(D)** 3750 **(E)** 4250

2010 AMC 12 A, Problem #11— "Because the sine of one angle is 0.6, we have a $3x-4x-5x$ right triangle."

Solution

Answer (D): Because the sine of one angle is 0.6, we have a $3x-4x-5x$ right triangle. The area of the triangle is $\frac{1}{2}$ times 60 times the length of the side corresponding to the altitude of length 60. To maximize the area this side should be the one measuring $5x$. In this case the area of the triangle is both $\frac{1}{2}\cdot 3x\cdot 4x$ and $\frac{1}{2}\cdot 60\cdot 5x$. Therefore $6x^2 = 150x$, $x = 25$, and thus the area is $\frac{1}{2}\cdot 60\cdot 125 = 3750$.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 9–12: analyze properties and determine attributes of two- and three-dimensional objects.